been incorporated into the later Christianity. The reformed Lāmaist Church of the Yellow-hat sect employ many rites which are transparently Bon, such as the necromantic expelling of the death-demon and the demons of sickness, and much of their divination-ritual. The unreformed Red-hat sects practise the old Bon rites to a much greater extent, including the erection of masts attached to dogs' and sheep's skulls, to 'bar the door' to the earth- and sky-demons.[1]

4. **Lāmaism and its sects and rites.**—The various aspects of Lāmaism and its sects and rites have already been described in previous articles.[2]

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L. A. WADDELL.

TIME.—1. **Introductory.**—Temporal characteristics are among the most fundamental in the objects of our experience, and therefore cannot be defined. We must start by admitting that we can in certain cases judge that one experienced event is later than another, in the same immediate way as we can judge that one seen object is to the right of another. A good example of the immediate judgment in question is when we hear a tune and judge that of two notes, both of which come in our specious present, one precedes the other. Another direct judgment about earlier and later is made in genuine memory. On these relations of before and after which we immediately recognize in certain objects of our experience all further knowledge of time is built.

It must be noticed that the relation, as given in experience, connects what we may call protensive events, *i.e.* events that have some duration, and not momentary events or moments. We are not directly aware of events without duration, still less of moments of empty time, and therefore are not directly aware of the relations between such objects. Momentary events, moments of time, and the relations which order them in a series are all known only after a long process of reflexion, abstraction, and intellectual construction. This does not necessarily imply that they do not exist in nature, still less that they are subjective and arbitrary; all that is meant at present is that they are not the objects of direct awareness. Again we must notice that the relations of before, after, and simultaneous with, as given in experience, are not mutually exclusive. Protensive events may very well overlap, and therefore we must recognize that the most general relation between them is that of partial precedence or consequence. Of course, when we become familiar with the conception of momentary events and see how convenient it is, we tend to define partial precedence in terms of them and their relation of total precedence. But the opposite direction must be followed if we want to start with the experienced facts and trace the logical development from them of the scientific notion of time. We must take the experienced relation of partial precedence as fundamental and define momentary events, moments, and the relation of total precedence in terms of partial precedence and events of finite duration.

That such a course is possible is shown by the fact that it has recently been followed to a satisfactory conclusion by Norbert Wiener in the *Transactions of the Cambridge Philosophical Society*, and by A. N. Whitehead in his *Principles of Natural Knowledge*. We may compare the duration of experienced events with the extension of visible and tangible objects, and the relation of partial precedence with the partial overlapping of two extended objects in the field of vision. The problem of defining momentary events, moments, and the serial relation of before and after in terms of protensive events and partial precedence is closely comparable to that of defining material points, geometrical points, and the relations of before and after on a straight line in terms of extended objects and their partial overlapping. The problem for time is, however, easier than that for space, because in the former we have only to deal with a relation that generates a one-dimensional series, whilst with the latter the experienced facts force us to define a three-dimensional manifold.

2. **Time and space.**—The analogy between time and space has long been recognized; and it will be useful to consider at this point just how far it goes and where it is supposed to break down. Let us consider the likenesses and the alleged differences.

(*a*) *Likenesses.*—(1) Most objects of immediate experience possess a kind of magnitude called extensity, and such objects stand in certain immediately recognizable relations to other objects of the same sense experienced along with themselves. Also the parts of any one such object have relations of this kind to each other. Similarly the objects of our experience have another kind of magnitude called protensity or duration. Such objects have to others of the same kind the relation of partial (or, in special cases, total) precedence, and this relation can be recognized immediately. Likewise the parts of a single specious present can be seen to have this relation to each other.

(2) The relations in each case have magnitude. Just as one object in the field of view can be more to the right of another than a third, so one event in the field of memory or in the specious present can precede another event by a longer interval than some third one.

(3) In each region there is the same close and peculiar connexion between the kind of magnitude possessed by the terms and the kind possessed by the relations. It is possible to say that 'the interval between two events A and B is as long as the duration of some event C, just as it is possible to say that the distance between two sticks laid in the same straight line is the same as the length of some third stick.

(4) It is commonly believed that, when the analysis is made into moments and momentary events, all the events in the history of the world fall into their places in a single series of moments. So too it is supposed that, when the analysis is made into material and geometrical points, all the points in the world take their places in a single three-dimensional series of geometrical points.[1]

(*b*) *Alleged differences.*—(1) It is commonly held that all events have temporal relations to each other, but that psychical events have no spatial relations. This is denied by a small number of philosophers, notably by Samuel Alexander.

[1] *Buddhism of Tibet*, p. 484 f.

[2] See artt. ABBOT (Tibetan); AṬIŚA; BHUTĀN, BUDDHISM IN; CELIBACY (Tibetan); CHARMS AND AMULETS (Tibetan); CHORTEN; DEATH AND DISPOSAL OF THE DEAD (Tibetan); DEMONS AND SPIRITS (Tibetan); DIVINATION (Buddhist); FESTIVALS AND FASTS (Tibetan); IMAGES AND IDOLS (Tibetan); INCARNATION (Tibetan); INITIATION (Tibetan); JEWEL (Buddhist); LĀMAISM; LOTUS (Indian); PADMASAÑBHAVA; PRAYER (Tibetan).

[1] We shall consider later what the Theory of Relativity has to say as to the impossibility of separating time and space and as to the notion of one single time-series.

Without questioning the possibility of correlating psychical events with positions in space, we must hold that this alleged difference is a genuine one. If in introspection we do contemplate our states of mind in the same sense as in perception we contemplate other objects, it seems clear that our states of mind show no trace of being extended or standing in spatial relations, but do have duration and stand in temporal ones. Alexander would, however, deny that we can contemplate our states of mind. If this be so, it would of course be quite possible that we should fail to become aware of the spatial characteristics of our mental states, even though they possess them; but of course we have no right to pass from this merely negative position to the conclusion that they actually do possess them. Alexander's positive reasons are bound up with a large and complex metaphysical theory into which we cannot here enter. In any case the present difference is merely an external one, and would not affect the essential similarity of space and time.

(2) A much more important point is that time is said essentially to involve the distinction between past, present, and future as well as that between before and after. Now nothing in space obviously corresponds to these distinctions in time.

(3) Closely connected with this alleged difference are a number of rather vague statements often made—*e.g.*, that parts of space co-exist, but that only the present moment exists.

These two supposed differences between space and time may be treated together. They rest largely on confusions into which it is very natural to fall. The distinction between past, present, and future is not one which, like that between before and after, lies wholly in the experienced objects, but is one that rests on the relations between experienced objects and the states of mind in which they are experienced. To begin with, the distinction between present and not-present at any rate may be usefully compared with that between here and elsewhere in space. Here means near my body; elsewhere means distant from my body. If we want an analogy to the distinction between past and future, we can find one in the distinction between things before and things behind our body. It is true, however, that this analogy is incomplete, and that for an important reason, though one extraneous to the nature of time. The reason is that our practical and cognitive relations towards the future are different from those towards the past. We know a part of the past at any rate directly by memory, but we know the future only indirectly by probable inference. There is no analogy to this in space; our knowledge of what is behind our body is of the same kind and of the same degree of certainty as our knowledge of what is in front of it. But we may imagine that a distinction like that between past and future would have arisen for space also, if we had been able to see straight in front of us but had never been able to turn our heads or our bodies round.

The distinction is sometimes drawn that the past is fixed and unalterable, while the future depends, in part at any rate, on our volitions. In what sense is this true?

Without involving ourselves in controversies about free will and determination, we may at least assume that the laws of logic apply to propositions about the future. Hence any proposition asserting the occurrence of any future event must be true or false, and cannot be both. In that sense the future is as determinate as the past. But two points have to be noticed. (i.) However much I may know about the laws of nature, I cannot make probable inferences from the future to the past, because I am not directly acquainted with the future, but I can make probable inferences from the past to the future; *i.e.*, although every possible proposition about the future is even now determinately true or false, I may be able to judge now, from my knowledge of the past and present and of the laws of nature, that some propositions about future events are much more likely to be true than others. (It must of course be remembered in this connexion that a proposition that is actually false may be much more likely to be true on my present information than one that is actually true.) (ii.) I know with regard to certain classes of events that such events never occur unless preceded by a desire for their occurrence, and that such desires are generally followed by the occurrence of the corresponding events. But the existence of a desire for x does not increase the probability that x *has* happened. If it did we might be said to affect the past in exactly the same sense in which we are said to affect the future. Thus the assertion that we can affect the future but not the past seems to come down to this: (*a*) that propositions about the future can be inferred to be highly probable from a knowledge of the past and present, but not conversely, because of our lack of direct acquaintance with the future; and (*b*) that the general laws connecting a desire for x with the occurrence of x always contain x as a consequent and never as an antecedent.

3. **Relation of time to logic.**—This brings us to the very important question of the relation of time to logic. If we say of any event e that it is present, this proposition will generally be false, and will be true only at one moment. It seems, then, as if the truth of the proposition altered with time. Any other proposition asserting the occurrence of an event—*e.g.*, Queen Anne is dead—seems to be equally at the mercy of time. Then again there seem to be other propositions that are totally independent of time—*e.g.*, $2 \times 2 = 4$. These are sometimes called eternal truths; they always state relations between universals, and all our *a priori* knowledge is of such propositions. Lastly, there are propositions which essentially involve time, but claim to apply to any time; *e.g.*, whenever it rains and I am out without my umbrella, I get wet. Thus, on the face of it, there seem to be three kinds of propositions as regards relation to time: (1) eternal truths, which are independent of time because they deal with the timeless relations of timeless objects; (2) hypotheticals asserting temporal relations between classes of events—these contain an essential reference to time, but not to any particular time; (3) propositions which assert the occurrence of particular events, and which *seem* to be true at certain times and false at all others, though this is not really so.

There are two points to notice about the last class of propositions. (*a*) All propositions about events essentially contain a reference to time, and all propositions about particular events essentially contain a reference to the particular time at which the event happens. This reference is not always made explicit; but, until this has been done, we cannot say that the verbal form stands for any definite proposition. (*b*) We have to distinguish between the time at which a judgment is made and the time involved in the proposition that is judged. When the latter is not made explicit in the verbal expression of the judgment, it is a convention of language to assume that the time in the proposition is intended to be that at which the judgment is asserted. Thus, if I say 'It is raining,' this verbal expression, since it clearly intends to refer to a particular event, is incomplete and stands for no definite proposition; for it says nothing about the time at which it rains. It therefore seems to be sometimes true and sometimes false. But, as actually asserted, the words would be taken to express my judgment of the proposition, 'It is raining at the time at which I say "It is raining."' And this proposition is timelessly true or false, subject to a further correction which we shall add in a moment. In fact, whenever we are told that a proposition is sometimes true and sometimes false, we know that we are dealing with an incomplete statement about an event, and that the real state of affairs is that a propositional function of the form 'e happens at t' gives true propositions for some values of t and false propositions for other values. But the propositions themselves are timelessly true or false.

It is important to notice that in practice there is always the possibility of any verbal statement about events, no matter how carefully put, being sometimes true and sometimes false. We are not directly aware of moments of time, and so can date events only by other events. And the persons who read or hear our verbal expressions may know only by some description the event which we use for purposes of dating. Now we can never be certain theoretically that only one unique event answers to any description however complicated, and often there is real ambiguity in practice. Take, *e.g.*, the amended expression offered above: 'It is raining at the time at which I say "It is raining."' To any reader of this article the expression remains ambiguous, because he knows the event that is used for dating only by the very ambiguous description, 'The writer's statement of the words "It is raining"'—a description which applies to dozens of different events. In practice the difficulty is solved in conversation by the fact that all the manifold circumstances under which the particular conversation takes place go into the description and make it practically unambiguous. In writing, the difficulty is solved practically by using as the origin of dates some event, such as the birth of Christ, whose full description is so complicated that it is almost certain that only one event answers to it. But the theoretical difficulty remains, and so we are tempted to say that any proposition about events is sometimes true and sometimes false. But the proper thing to say is that any verbal expression referring to events, no matter how carefully put, always runs a theoretical risk of ambiguity—*i.e.*, it might with equal propriety make one reader think of one proposition which is true, and another of another proposition which is false.

We can now apply these general results to the special case of events being sometimes future, and then present, and finally past. The statement '*e* is present' is essentially incomplete and ambiguous, for, as we loosely say, it is sometimes true and sometimes false. The first thing, then, is to fill in the special time involved in the proposition. We then get '*e* is present at *t*,' where *t* is some definite moment fixed by some system of dating from a well-known and presumably unique event. What does this statement mean? Assuming that there are such things as moments, it means that *e* is at the moment *t* in an analogous way to that in which an object is at a position in space. The statement '*e* is present at *t*' may be compared with the statement 'Mr. Asquith is present at the meeting,' which means that his body is in the place where the meeting is held. In all complete statements of the form '*e* is at *t*' we must understand the word 'is' as standing for a timeless copula, and distinguish it from the 'is' of the present tense, which is contrasted with 'was' and 'will be.' Let us denote the 'is' of the present tense by 'is now.' Then the statement '*e* is now present' is an incomplete statement which is interpreted in use to mean '*e* is at (or occupies) the same moment as my assertion that it is now present'; '*e* is now past'='*e* was present'='*e* is at a moment earlier than my assertion that *e* is now past.' Similarly, '*e* will be present'='*e* is now future'='*e* is at a moment subsequent to my statement that *e* is now future.' The laws of logic are of course concerned with the timeless copula, and they presuppose that statements containing tenses are reduced in the way suggested above.

4. **Past, present, and future.**—We see, then, that the real source of the distinction between past, present, and future, and of the difference here between time and space, is that our judgments as well as the events judged about are in time, whilst our judgments about things in space are not in any obvious sense in space. These three distinctions correspond to the three possible temporal relations between our judgments and the events which our judgments are about. These distinctions are important, and they have been enshrined in language because they are correlated with important epistemological and psychological differences. Some states of mind are essentially contemporary with their objects—*e.g.*, the immediate awareness of visual sense-data when I open my eyes.[1] Other states are essentially later than their objects—*e.g.*, memories. If we exclude the possibility of prophecy, we may state the important epistemological proposition that all states of mind which give us an immediate knowledge of existents are either contemporary with, or later than, their objects.

It is important to notice that these statements are not merely analytic. There is a psychical difference between memories and awarenesses of contemporary sense-data which is open to introspection (though, of course, there may be marginal cases where the difference falls below the threshold of distinguishability), so that the statement that the former succeed and the latter are contemporary with their objects is a synthetic proposition.

We must, moreover, take into account the facts described in psychology as the specious present. In the first place, we must say that, if an object be known directly by a state of mind which succeeds it by more than a certain short time *t*, which seems to be fairly constant for a given individual, the state counts introspectively as a memory, and the object is judged to be past. If the period between the object and the direct awareness of it be not greater than *t*, the awareness does not count for introspection as a memory, and the object is judged to be present. To say, then, that an object has been present and is now past means that (*a*) it is (timelessly) the object of an immediate awareness which succeeds it by less than *t*, and (*b*) that my statement 'It is now past' succeeds it by more than *t*. We have still, however, to consider what is meant by the presentness of a state of mind. This seems to mean that, if a state of mind be the object of an act of introspection which succeeds it by less than a certain short period, the state presents a certain peculiar characteristic which it does not present to any later act of introspection.

We can now deal with such statements as that only the present exists, or that the present is a mere transition from one infinite non-existent to another. These phrases are mere rhetoric rooted in confusions. It is perfectly true, of course, that the whole history of the world is not a complex of co-existing parts (in the sense of parts existing at the same time), as a table is. But this does not mean that it is not a whole, or that one part of it exists any less than any other part. To say that *x* no longer exists, or does not yet exist, simply means that it occupies a moment before or after my statement about it. At another moment I may make another statement of the same verbal form about *x*, and, since this no longer stands for the same proposition, it may no longer be true (*i.e.* no longer stand for a true proposition). But this involves no change in *x* itself. That *x* exists at a certain moment simply means that *x* occupies that moment, and this is timelessly true. Similarly, the fact that this moment has a certain temporal relation to any definite assertion that I may make about *x* is timelessly a fact. That it has different and incompatible temporal relations to various assertions of the same verbal form made by me is also timelessly true, and is not merely compatible with but also a necessary consequence of *x*'s existence at its own moment. An event must continue to *be*, if it is to continue to stand in relations; the battle of Hastings continues to precede the battle of Waterloo, and therefore both these events must eternally be at their own respective moments. That both have ceased to be present merely means that they precede any assertion that I or my contemporaries can make about them; that both were once present merely means that both are contemporary with some assertions made about them.

[1] It is better for the present not to call these states of mind either perceptions or sensations, because the object of a perception is generally supposed to be a physical object or its state, and this may exist millions of years before the perception—*e.g.*, the perception of a distant star. Similarly, to call these states of mind sensations would lead to misunderstandings, owing to the ambiguities of that word and the widely held belief that sensations do not have objects.

The fallacy which we have to avoid is that of confusing two different senses of co-existence. In one sense the parts of any related whole co-exist; in another only those events that occupy the same moment of time co-exist. It is clear that the whole course of history does not co-exist in the second sense, and it is thought that this prevents it from co-existing in the first. Yet this is necessarily false, since it is admitted that events do have and continue to have temporal relations, and therefore they must form a related whole all of whose parts have being. The confusion is increased by the belief that past, present, and future are essential characteristics of objects in time in the same way as before and after are, instead of being analysable into the temporal relations of states of mind and their objects.[1]

When it is once recognized that the whole course of events is in a certain sense a *totum simul*, it becomes easy to see the answer to the famous theological problem: How can God's foreknowledge of men's actions be compatible with the freedom of men's wills? The answer is as follows. Whether men's wills be free or not, every man's future actions are as completely determinate as his past ones; this is a mere consequence of the laws of logic. If indeterminism be true, then no amount of knowledge about events previous to a moment t, and about the general laws of nature or the particular habits of a man, will enable us or even God to infer with certainty what the man's volition at t actually is, although it is eternally perfectly determinate. These two statements are clearly quite compatible. Finally, in spite of the fact that God cannot *infer* the man's volition at t, He may at any and every moment be directly aware of it in precisely the same way as we are aware directly (and not merely inferentially) of certain events through memories which are themselves later events. The facts that at a certain moment t_1 God can have a state of mind whose immediate object is the volition of a man at some later moment t_2, and that no amount of knowledge of events before t_2 would enable Him to *infer* the volition at t_2, are perfectly compatible; and they cease to be even paradoxical when we compare the case of memory and note that there is no essential difference between past, present, and future.[2]

5. Reality of time.—A great many philosophers have been concerned to deny the reality of time. Their arguments fall into two groups: (1) those that depend on the supposed infinity and continuity of time, and are therefore equally applicable to space; and (2) those that depend on the supposed peculiarities of time—*e.g.*, on the distinction of past, present, and future. Before considering the arguments in detail, it will be useful to make some quite general reflexions.

(i.) It is a matter of direct inspection that the immediate objects of some of our states of mind have temporal characteristics. It is as certain that one note in a heard melody is after another in the same specious present and that each has some duration as that some objects in my field of view are red or square and to the right or left of each other. It is then quite certain that *some* objects in the world have temporal characteristics, viz. the immediate objects of some states of mind. Now it is also certain that these objects exist at least as long as I am aware of them, for, in such cases, I am obviously not aware of *nothing*. Hence there cannot be anything self-contradictory in the temporal characteristics found in these objects, for otherwise we should have to admit the existence of objects with incompatible characteristics. Hence there is no obvious reason why temporal characteristics should not also apply to what is not the immediate object of any state of mind. It follows, then, that criticism cannot reasonably be directed against temporal characteristics as such, but only against the descriptions that we give of the temporal characteristics of experienced objects, and the conclusions that we draw from them or the constructions that we base on them. And arguments that refer to the infinity and continuity of time are really directed against a construction based on what we conceive to be the essential characteristics of the time element which is undoubtedly present in the objects that we experience; for we are not directly aware of infinite duration or of the continuity—in the mathematical sense—of time. If we suppose that such criticisms are successful, the conclusion ought not to be either that reality has no temporal characteristics (for it is quite certain that at least some parts of it have), or that time, as an inference or construction extending the temporal characteristics of experienced objects to others, is unreal (for this goes much too far). The only justifiable conclusion would be that one particular way of describing and extending the temporal characteristics of experienced objects is unsatisfactory, and that it behoves us to look for a better one. This point has not commonly been grasped by philosophers who claimed to disprove the reality of time.

(ii.) It is thus obviously of importance to be clear as to what is the particular view of time that is attacked by special arguments. The important distinction for us to make is this: it is possible to hold (*a*) that there is a series of moments of time, and that events occupy some of them but are distinct from them, and have temporal relations to each other in virtue of those which subsist between the moments that they occupy; or (*b*) that there are no such things as moments distinct from events, but that events really do have direct temporal relations to each other; or (*c*) that there are no moments, and that even events only *appear* to have temporal relations to each other. It is clearly possible to deny (*a*) without denying (*b*). To do this can hardly be called denying the reality of time; it should rather be called denying the absolute theory of time in favour of the relative theory. It is only philosophers who deny both (*a*) and (*b*) and support (*c*) who can strictly be said to deny the reality of time. It is quite possible, however, that some arguments might be equally fatal to (*a*) and to (*b*).

It will be well at this point to say what we can about the controversy between absolutists and relativists. The absolute theory strictly means that temporal relations between events are regarded as compounded out of two relations—(1) that of an event to the moment of time which it occupies, and (2) the relation of before and after between moments of time. The relative theory holds that there are no moments, but that temporal relations hold directly between events. Its most important philosophical upholder is Leibniz, though he goes a good way farther in the direction of (*c*); it is also held, with a good deal of misunderstanding and confusion, by many modern physicists of a philosophical bent. We may say that the relative theory stands at one remove, and the absolute theory at two removes, from what we find in the objects of immediate experience. Here we find, as we have seen, events of finite duration and relations of partial precedence. The relative theory replaces these objects by series of momentary events of no duration, and the relations by those of total precedence and simultaneity. The absolute theory takes the farther step of introducing a new set of entities, viz. moments which have no duration and stand in relations of total precedence but never of simultaneity, and a new relation, viz. that between a momentary event and the moment which it occupies.

Neither theory has been very accurately stated by most of its supporters; *e.g.*, Newton, the chief upholder of the absolute theory, was mainly concerned with the measurement of time and the desire for a constant rate-measurer. But the two theories, when thought out, may be reduced to what has been stated above. We may say at once that we know of no way of deciding conclusively between the two. But, although moments and momentary states *may* exist, we now know that all their

[1] The point can perhaps be made clearer by reflecting that a tune has a pattern in time in exactly the same sense as a wall-paper has a pattern in space.

[2] We can, of course, remember much that we could not infer.

work can be done by certain logical functions of nothing but events of finite duration and their relations of partial precedence. Hence both theories may be said to sin by assuming entities which are not necessary to science and cannot be either directly or indirectly verified (viz. momentary events in the relative theory, and moments in the absolute theory), and the absolute theory is the worse sinner of the two. As certain logical functions of what actually exists (viz. certain classes of classes of events), moments do exist; but whether there also exists anything having the same logical relations but of the type of individuals and not of that of classes of classes it seems totally impossible to determine. It is, however, often convenient to continue to speak in terms of moments, and this is harmless for the reasons given above.

We can now deal with the special arguments against time.

(1) Those based on its supposed infinity commonly confuse infinity with endlessness. They generally proceed on the assumption that what is meant by the infinity of time is that it has neither a first nor a last moment. But this would be perfectly compatible with the whole course of time lasting for no more than a second. The fractions between 0 and 1, arranged in order of magnitude, have neither a first nor a last term, and yet the interval between any two of them is less than unity. But all attempts to prove that time or the series of events must have an end fail. So do attempts to prove that they cannot have ends. The most celebrated argument on both sides of this question is contained in Kant's first antinomy. His argument against the endlessness of time, interpreted as charitably as possible, comes to the statement that, because there are definite points in the time-series—in particular, the point which we have reached when we read Kant's argument—therefore the series must have a definite beginning point. Otherwise, Kant says, the series of events could never have reached the definite point which it admittedly has reached. The argument is, of course, a complete *non-sequitur*, for it practically amounts to saying that a series cannot have *any* definite term unless it has *end* points. And this is sufficiently refuted by considering that the number $+2$ is perfectly definite, although the series of numbers with signs has neither a first nor a last term.

Arguments to prove that time or the series of events in time cannot have a beginning are perhaps more plausible. It is difficult for us psychologically to imagine a first event or a first moment, because all the events that we can remember have been preceded by others. Also there are special difficulties connected with causation in the notion of a first event, which do not apply to a first moment or to a last event. A first event is one which no event precedes, though there may of course be moments that precede a first event. Now, the only plausible general proposition about causation seems to be that, if the whole universe were completely quiescent for a finite time, it could not begin to change.[1] This means that, if the universe be in the same state at any two moments t_1 and t_2 and at all moments between them, it will be in the same state at all moments later than t_1. Now, to say that a change happens at t_1 means that, if the state of the universe at t_1 be s_1, and if it also be s_1 at any later moment t_2, then there is a moment between t_1 and t_2 at which its state is different from s_1. It follows from this definition that to say that a first event happens at t involves that the universe has been in the same state for a finite time before t. And this is contrary to our proposition about causation. If, then, we accept this proposition as an *a priori* truth, there cannot be a first event, though there might be a first event in certain isolated parts of the universe (*e.g.*, the creation of the world) provided that there had never been a first event in other parts (*e.g.*, in the mind of God). But, of course, there remains the doubt whether our axiom about causation be not a mere prejudice masquerading as an *a priori* law.

[1] The universe here must be taken to include God, if there be one.

It must be carefully noted that, if there be a first event, there need not be a first moment of time, and that, if there be a first moment of time, there need not be a first event. Again, if there be a first moment of time and no first event, either there might be no moment, except the first, that was not occupied by an event, or there might be a duration unoccupied by events. These consequences follow from the continuity of time, and have often been overlooked by philosophers ignorant of the mathematical theory of continuity.

Leibniz based his main argument against the absolute theory of time on the fact that, if it were true, there might be a period, finite or infinite, before any event happened. This period must be definite; and yet, the moments of time having no intrinsic difference, there is no reason why it should be ended or limited by one moment rather than another. If, on the other hand, we avoid this by assuming that there is an event at every moment of time, there is no reason for assuming *both* events and moments, for the series of events will suffice.

This argument is a sound one against assuming that there *are* moments, though it certainly cannot disprove that there may be moments. If there were moments, they would doubtless have intrinsic differences, though we could not discover them; we must further recognise some ultimate facts, and one of these might be that the course of events is preceded by such and such a duration of empty time.

We may sum up our conclusions as follows. Arguments to disprove the reality of time from its infinity and continuity either confine themselves to criticizing infinity and continuity as such or introduce considerations about causality. Arguments of the first kind would be equally fatal to *any* infinite or continuous series, and therefore prove too much, for they would destroy the series of real numbers. And we now know that all such arguments do rest on confusions and on an insufficient analysis of the notions of infinity and continuity. There is therefore no reason why the series of moments at any rate should not be either (*a*) endless or (*b*) of infinite length. The second set of arguments can apply only to events and not to the supposed series of moments, because causation is concerned with events and not with empty time. We saw that, if a certain plausible axiom about change be true, there cannot be a first event. This would not, however, prove that the whole series of events has lasted for an infinite time, though the present writer knows of no objection to such a possibility. There is no more objection to the series of events being endless than to any other series being endless—*i.e.*, there is none at all. The result is that all danger of a valid antinomy against time vanishes. (i.) Whether the axiom about change be true or not, it is equally possible that the series of moments shall be (*a*) endless or terminated, and (*b*) of finite or of infinite length. (ii.) If the axiom about change be true, the series of events cannot have a beginning, but may (*a*) have an end or not, and (*b*) be of finite or of infinite length.

(2) Arguments against the reality of time which turn on the distinction of past, present, and future may be dealt with shortly. One argument asserts that the past and the future do not exist, and that the present is a mere point without duration. It is then supposed that what occupies no finite duration cannot be real, and this disposes of the present. An argument of this kind is used

by Leibniz against absolute time, though it would presumably apply to events just as well. It is met, of course, by the consideration that past, present, and future are all always equally real, and that these characteristics do not belong to events as such, but in virtue of the temporal relations between them and certain psychical events.

A somewhat different argument against the reality of time has been produced by J. M. E. McTaggart.[1] His argument is that every event is past, present, and future; and that the attempt to avoid the incompatibility of these predicates by saying that the event has been future, is present, and will be past involves a vicious circle or a vicious infinite regress. The answer is that, whenever we consider any definite statement about the pastness, presentness, or futurity of an event, we can see that there is no contradiction. Take a definite statement by McTaggart that Queen Anne's death is now past and has been present and future. Suppose we interpret this to mean that Queen Anne's death is not the direct object of any awareness (even a memory) which is contemporary with McTaggart's statement, but that it is contemporary with some states of mind (*e.g.*, Lord Bolingbroke's) which precede McTaggart's statement; and that it is later than some thoughts about it (*e.g.*, William III.'s), which also precede the statement. Then those three propositions seem to be timelessly true, perfectly compatible, and to contain all that is meant in the assertion by McTaggart that Queen Anne's death is past and has been present and future.

We may conclude, then, that no satisfactory proof has been offered even that absolute time is unreal, still less that the series of events and their direct temporal relations are unreal.

6. Measurement of duration.—It seems to have been the question of a rate-measurer that led Newton to the theory of absolute time. Newton considers a number of periodic events which are roughly isochronous, and compares their rough isochronism with 'absolute time, which flows uniformly.' It is an unfortunate way of introducing absolute time. In the first place, it is of no practical use to any one. Whether absolute time flows uniformly or not, we can only observe events and must use them, or processes based on them, as our rate-measurers. Again, the statement that absolute time flows uniformly is thoroughly obscure. Time cannot be said to flow, for this seems to imply that time changes; and this would make time consist of a series of events in time. Nor is it at all clear what Newton meant by uniformity in this connexion. Presumably the meaning must be that the moments of time form a series like the real numbers. What we really want to know is whether we can find any periodic process such that the time that elapses between corresponding stages in each repetition is the same. But no essential reference to absolute time is involved here. We must beware of confusing the two statements: (1) there are definite intervals of a certain determinate duration, and this duration is independent of our methods of measurement; and (2) there are absolute moments of time, and the interval between any two of these has a definite magnitude. The latter implies the former, but not conversely. The real problem is: Granted that there is a definite interval between pairs of events, how are we to measure it?

There is a special difficulty in measuring intervals of time between events which is not nearly so much felt in measuring the distance between things. This difficulty is in the temporal analogue to superposition. We may carry a rod about with us in space, and we may have fairly good reasons to believe that it has not altered in length. The corresponding procedure in time-measurement is to find some process which can be started and stopped at any moment and can be assumed to have the same period whenever it is repeated. Such processes may be called isochronous. But, even when an isochronous process has been secured, it cannot be used to measure time in the same direct way in which a rod can be used to measure length. A rod will not as a rule fit an exact number of times into what we want to measure; it is therefore divided into a number of equal parts. Similarly we want an isochronous process that can be divided into equal subdivisions which can be easily recognized; *i.e.*, we want a process which itself consists of a number of similar processes which all occupy equal times. Now, it is not nearly so easy to be sure that a process takes the same time whenever it is repeated as to be sure that a rod keeps the same length wherever we use it; and it is much less easy to divide a process into parts that occupy equal times than to divide a rod into parts that have equal lengths. The recognizable divisions in a process of change are largely fixed for us, while divisions on a rod can be fixed by us with marks without affecting the rod as a whole.

Nevertheless the assumptions that have to be made, and the peculiar mixture of observation and convention that is involved, are the same in principle for the measurement of time and of space. The fact is that we can make immediate comparisons both of length and of time with a certain amount of accuracy. We believe that these judgments are the more accurate the nearer the objects to be compared are in time and space, and the more similar the circumstances under which each is inspected. Trusting to these immediate judgments, we see reason to believe that both the lengths of rods and the time taken by processes may vary when the rods are moved or the processes repeated. But we believe that the variation always depends on the fact that change of position in space or time involves change in the relations of the rod or the process to pieces of matter, and that mere changes of position in absolute time and space—if such could be—make no difference. We have learned by experience what are the most important factors that determine change of length or of period, and we can allow for them. It is found that the periods of recurrent processes are, on the whole, more largely affected by changes in the surroundings than are the lengths of such bodies as steel rods.

Our method of determining an ideal rate-measurer is somewhat as follows. We begin with some process which is sensibly isochronous—*e.g.*, the swing of a pendulum, or the time taken for a complete rotation of the earth on its axis. We can judge of this isochronism with a certain amount of accuracy by direct comparison in memory, just as we can compare lengths by looking at them. We can go farther than this. Just as we are greatly helped in our comparison of lengths by putting the objects to be compared side by side, so we can use expedients to help our judgments of the isochronism of processes. If we start two pendulums together and their periods be not exactly the same, the divergence will become more and more marked the longer they swing. If no divergence be noted after many swings, we may conclude that each swing of one takes the same time as the corresponding swing of the other. This does not prove that the successive swings of either are isochronous; for the period of each may be varying according to the same law. But, if we also find that the period of one of these processes synchronizes with the corresponding period of some other

[1] 'The Unreality of Time,' *Mind*, new ser., xvii. [1908] 457-474.

sensibly isochronous process which is physically very different, it becomes very improbable that there should be any law by which the successive periods of two such very different processes alter in precisely the same way. We are therefore justified in concluding *tentatively* that the successive periods of these sensibly isochronous processes are actually isochronous.

The next step is to state all the laws of nature which involve time on the assumption that equal intervals of time are measured by complete periods of such processes. We find, *e.g.*, that, if it be supposed that the successive rotations of the earth on its axis are isochronous, the laws of motion can be very simply stated and are very nearly verified by all the mechanical phenomena that we can observe. So far we are entirely in the region of what can be experienced or rendered very probable from what we experience. But now a conventional element enters. We shall probably find that, when time is measured by an actual physical process and when our laws have been stated in terms of time so measured, a closer investigation shows that there are slight divergences from the laws which cannot be accounted for by mere experimental errors. The last stage in the determination of the equality of times now begins. We argue that the suggested laws are so simple and so nearly true that the most reasonable plan is not to keep the same time-measures and complicate the laws, but to suppose that the laws are rigidly true but the time-measurer not perfectly accurate; *i.e.*, that successive periods of this physical process are not perfectly isochronous. We therefore erect the laws into principles, define equality of times by them, and apply the necessary corrections to our old time-measurer. There is nothing particularly arbitrary about this. We believed, to begin with, as the result of direct judgments assisted by the use of such expedients as have been described above, that a certain periodic process is isochronous. We admitted, however, that deviations from isochronism so small as to escape the notice of any direct method are possible. We then stated our laws in terms of time as measured by this process, and found them to be simple and very nearly true; but, if they are to retain their simple form and become quite true, a small correction must be made in the assumed isochronism of the process. This contradicts nothing that we have deduced from our experience; for we admitted all along the possibility of errors too small for direct detection. The procedure has the least trace of arbitrariness if, as is often the case, we can see the physical cause of the lack of complete isochronism in our time-measurer and can fully explain this lack in accordance with the laws which we have erected into principles. This has happened, *e.g.*, with the earth as a time-measurer, where we can explain its small defect from isochronism, when once we have to assume it, by the frictional effect of the tides acting according to the laws of mechanics. Even when no physical cause can be detected for the presumed lack of isochronism, it is always possible to suggest a hypothetical one. But, in so far as this has to be done, our procedure does become more arbitrary; and a point may be reached where a full explanation of all the phenomena demands a real change in the form of the laws with or without a change in the time-measurer. This has happened in recent years to the laws of motion, mainly through investigations on the movements of small electrically charged particles with a velocity comparable to that of light.

7. Theory of Relativity.—The next point to be considered is the criterion of beforeness, afterness, and simultaneity among events which are not the objects of any one experience. We have seen that in favourable cases we can immediately judge that one event that we experience is after another that we experience. Other people can make similar judgments about events in their experience. But we cannot directly judge of the temporal relations of events which we do not directly experience. Matters are on exactly the same footing with spatial relations. I may be immediately aware that one object in my field of view is to the right of another in the same field, and another man may be able to make similar judgments about his visual sense-data. But it remains to be seen what is meant by saying that an object which A experiences is at the right of one which B experiences; or again what is meant by the statement that of two objects which no one experiences—*e.g.*, two atoms—one is to the right of the other. What is wanted is to be able to date events in a time-series which is neutral as between A's experience and B's, and shall contain events that do not fall into the direct experience of any one. In this problem we must carefully distinguish between two questions which are liable to be confused: (1) How do we come to understand the nature of the relations in the neutral time-series?, (2) How do we know with regard to any two definite events, e_1 and e_2, whether e_1 is before or after or simultaneous with e_2?

The answer to the first question is that the relations in the neutral series are regarded as having the same logical properties as those which we directly experience, or at any rate as being capable of definition in terms of the logical properties of these relations. Possibly a temporal relation as experienced by A has a sensuous particularity different from that possessed by one experienced by B; just as it is impossible to say whether the quality of what A sees and that of what B sees are precisely the same when they say that they perceive the same colour, and no available test can detect any discordance between their experiences. But, of course, the sensuous particularity is what is shed when we consider a neutral time-series, and only the logical properties of the relations (*e.g.*, transitivity, asymmetry, etc.) are important.

The distinction between the space and time of each man's experience and a neutral space and time runs parallel with the distinction between the immediate objects of each man's experience and neutral (or, as we call them, physical) objects. However we suppose physical objects to be constituted, and whatever we suppose to be the relation between our minds and them, it must be assumed that physical objects are in the neutral space, and that their changes take place in the neutral time and make themselves known to us by correlated changes in the immediate objects of our experience.

It is not necessary here to consider how a number of people, $M_1 \ldots M_n$, come to agree that certain events, $e_1 \ldots e_n$, in their respective sense-data are all correlated with the same physical event. But it is necessary to notice that they will find, first of all in the case of sound, that, if their physical laws are to give at all a simple and complete account of what they may expect to hear under given circumstances, they must assume that the sounds heard by various people, and all correlated by them with a single physical event, are not in general contemporary with each other. The greatest accuracy and simplicity is introduced into the laws of sound by supposing that the hearing of the sounds by the various people takes place at times dependent on the positions of their bodies in physical space and on the spatio-temporal position of the single physical event correlated with all these sounds. This example brings out three very important points. (1) The determination of the temporal relations between events in the minds or in the immediate objects of the minds of different people can be accomplished only when these events have been correlated in some definite way with supposed neutral physical events; (2) the temporal relations then assigned are such as to make the laws telling us what sensations to expect in given circumstances as simple and accurate as possible; (3) it follows from these considerations that the determination of a neutral time-series and of the positions of physical objects in a neutral space must proceed *pari passu*.

Suppose, *e.g.*, that we say that the velocity of sound is v centimetres per second: (1) we want to connect all the known facts about the sounds which people hear under circumstances that can be directly experienced; (2) we want to do this compatibly with the assumptions which have already been made as to what heard sounds are to be classed together as connected with one physical event; and (3) we want our laws which sum up the known facts and anticipate experience to be as simple as is compatible with accuracy. We find that these ends can best be accomplished by supposing that *A*'s hearing of s_a and *B*'s hearing of s_b (s_a and s_b being both correlated with the single physical event *S*) take place at times $t+\frac{s_a}{v}$ and $t+\frac{s_b}{v}$ respectively, where t is the date of *S* in the neutral time-series, and s_a and s_b are the respective distances between the place where *S* happens in physical space and *A*'s and *B*'s bodies as physical objects. We must remember that the correlation of several sounds heard by different people with a single physical event and the assignment of positions in neutral space to physical events are themselves carried out on the same general principles as the dating of events in neutral time and as the measurement of duration already described; *i.e.*, we start with instinctive judgments of rough accuracy, and then proceed to a more accurate determination of our terms, guided by the general motive of maximising the accuracy and simplicity of scientific laws.

As we have seen, sound is the first and most obvious case where it is necessary to assume different dates for different members of a group of sense-data which are all correlated with a single physical event. The more accurate researches of science necessitate a similar process for dealing with the sense-data of sight, and so the notion of a velocity of light is introduced. These velocities, once determined, furnish a criterion of before and after among physical events, and, through them, for events in different minds.

Let us denote any moment at *A* by the symbol ${}^a t_r$, and an event which happens at the point *A* at the moment ${}^a t_r$ by ${}^a e_r$. Let us use the same notation for events and moments at *B*. Then we can say: An event ${}^a e_r$ precedes an event ${}^b e_s$ if a disturbance leaving *A* at ${}^a t_r$ reaches *B* not later than ${}^b t_s$. Now it is found that we have no reason to believe that any disturbance travels faster than light. It can be shown that, if the above be our sole criterion for before and after between events at different places, there will be pairs of such events of which we have no reason to say that one is either before, after, or contemporary with the other.

To see this, consider the following case. Let ${}^a e_1$ happen at *A* at ${}^a t_1$. A signal which leaves *A* at ${}^a t_1$ cannot reach *B before* a certain moment ${}^b t_2$. Again, a signal that reaches *A* at ${}^a t_1$ cannot have left *B after* a certain moment ${}^b t_0$. On our criterion, therefore, ${}^a t_1$ is before any moment that is after ${}^b t_2$ and is after any moment that is before ${}^b t_0$. But how are events at *B* which happen between ${}^b t_0$ and ${}^b t_2$ related in time to the event ${}^a e_1$? Take an event ${}^b e_s$ such that ${}^b t_s$ is between ${}^b t_0$ and ${}^b t_2$. You cannot say that it is before ${}^a t_1$; for a disturbance leaving *B* at ${}^b t_s$ would reach *A* later than ${}^a t_1$. But you also cannot say that ${}^a t_1$ is before it; for a disturbance leaving *A* at ${}^a t_1$ would reach *B* later than ${}^b t_s$ (viz. at ${}^b t_2$). Hence on our criterion we can neither say that ${}^b e_s$ is before ${}^a e_1$ nor that ${}^a e_1$ is before ${}^b e_s$. Moreover, there is an infinite number of events at *B* of the form ${}^b e_s$ where s is between 0 and 2. Thus we cannot cut the knot by saying that, since they are neither before nor after ${}^a e_1$, they are contemporary with it. For they are not contemporary with each other. Thus one and only one of the class of events ${}^b e_s$ can be taken to be contemporary with ${}^a e_1$, and the rest, so far as our criterion goes, must be held to be neither before, after, nor simultaneous with ${}^a e_1$. We are thus compelled to recognise that we may have no means of deciding whether a pair of events at different places in physical space are contemporary or not in physical time.

We can, if we like, accept this result, and build up our physics on the assumption that physical time really is non-connexive; *i.e.*, that, though all events have temporal relations to some events, none have temporal relations to all events. This has recently been done very fully and ably by A. A. Robb.[1] Or we may take the more usual course of assuming that physical time really is connexive, but that in certain cases all criteria fail to determine the actual temporal relations which subsist between events in different places. We then must simply make a convention (to return to our example) that one particular event of the class of events at *B*, whose temporal relations to ${}^a e_1$ are left doubtful by our criteria, is contemporary with ${}^a e_1$, and that whatever precedes this one precedes ${}^a e_1$ and whatever follows it follows ${}^a e_1$. It is customary to assume that the event at *B* which comes midway between ${}^b e_0$ and ${}^b e_2$ is contemporary with ${}^a e_1$; but it must be noticed that this is a mere convention, though doubtless the most reasonable one to make. (On our notation this event would naturally be ${}^b e_1$.)

We must notice further that, for this convention to be determinate at all, we must assume that we know that the time-measurer at *B* goes at the same rate as that at *A*, and that both go uniformly. Now, if the time-measurers cannot be moved about, their synchronism can be determined only by sending signals from one to the other—*e.g.*, light-signals. And, even if they can be moved about, our only test for the continuance of their synchronism, when they have been moved apart and are no longer in view together, is by light-signals. On the other hand, the question whether our tests for synchronism by light-signals are genuine tests (*i.e.* involve synchronism in physical time) depends on whether the velocity of light relative to the system containing the time-measurers is constant in time and the same in all directions. And this last point cannot be determined *until* the time-measurers in two places have been synchronized; for it is obvious that to measure a velocity we need to know the time in two places. We see, then, that the possibility of synchronizing time-measurers and the uniformity of the velocity of light stand and fall together, and that neither can be proved independently of the other. If we allow that the velocity of light relative to the system is constant and uniform in all directions, our tests for synchronism and uniformity in our time-measurers are valid; if we allow that the criteria ensure physical synchronism, the physical velocity of light (as distinct from its *numerical* measure on our convention) will be constant and uniform. But neither question is or ever will be capable of independent settlement; and therefore we simply have to make a convention that the meaning to be attached to synchronism in different places is agreement with the tests based on light-signals, and another convention that distances shall be so measured that the measure of the velocity of light relative to the system is independent of time and of direction.

Suppose now that the people on a system *S* determine their spatio-temporal co-ordinates in this way, and that the people on another system S^1, moving with uniform translational velocity relative to *S*, determine their spatio-temporal co-ordinates similarly. Let them arrange, as they can do, that the time-measurers at the origin of each go at the same rate; and further let them arrange their units so that the velocity of light as measured by each from experiments with sources and mirrors fixed in their own system shall have the same numerical measure. Then (*a*) it can be proved that each will find the same numerical measure for the velocity of light, even though the sources and mirrors be in uniform motion relative

[1] *A Theory of Time and Space.*

to the two systems. (*b*) It is possible to find equations connecting the spatio-temporal co-ordinates which the people on S give to any momentary event which they observe with those which the people on S^1 give to the same event. These are the celebrated transformations of the Theory of Relativity. They are, as we should expect, perfectly reciprocal, since the relative motion of S and S^1 is a perfectly mutual phenomenon. But (*c*) they lead to certain rather startling results. (1) Lengths along and at right angles to the direction of relative motion which are judged to be equal by the people on one system will be judged to be unequal by those on the other. The ratio depends on the relative velocity and on the value of the velocity of light which is common to the two systems.[1] (2) Events in different places which are judged to be contemporary by the people on one system will be judged to occur at different times by those on the other system, and the difference of time will depend on the distance apart parallel to the direction of relative motion.

Although the observers on the two systems thus differ, they cannot criticize each other. Each has pursued precisely the same plan in setting out his co-ordinates and synchronizing his time-measurers. And it would be quite futile for one to claim that his results are the right ones because his system is at rest and the other is in motion. For the relative motion is completely reciprocal, and neither absolute motion nor any consequence of it can be observed. Lastly, it is equally futile for one to say that he is at rest 'relative to the ether,' while the other is in motion; for we know that no experiment whatever has been able to demonstrate motion 'relative to the ether,' and this motion may fairly be dismissed as a fiction. The upshot of the matter is that there is nothing to choose between their respective judgments, and that all the laws of nature can be stated as truly and will have precisely the same form, no matter which of an infinite number of systems in uniform translational motion be taken as the basis for spatio-temporal co-ordinates. This result, with the mathematical consequences that flow from it, is known as the Theory of Relativity. Its philosophical importance is that it enables us to see the tacit assumptions that are made when we talk of events at different places being contemporary; and the fact that measurement of distance is entangled with time, since the distance between two objects at any time involves a decision as to what is meant by the same time in two different places. Though it no more completely refutes the possibility of absolute space and time than does any other argument (for after all it only deals with our numerical measures and leaves it open whether one system of time-measurers is physically uniform and synchronous and one system of space-measures directly represents distances in physical space), yet it helps to render the notions of absolute space and time still more spectral and remote from all possible experience than before. For it enables us to see that there are a certain indeterminateness and conventionality even in the measurement of the distance between physical objects and of the lapse between events; and that therefore what we can know is even at a farther remove than we had thought from the points of absolute space and the moments of absolute time.

The Theory of Relativity sketched above was first fully stated by Einstein in his classical paper, 'Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen,' which appeared in the *Jahrbuch der Radioaktivität und Elektronik* for 1907. This may be called the restricted Theory of Relativity. It may be briefly characterized as consisting of an experimental fact and a philosophical principle suggested by a great number of facts. The philosophical principle is that, since we can never observe absolute time, space, or motion, even if there be such things, the laws o physical phenomena as learned from experiment and observation must retain the same *form* for acts of observers in uniform motion relative to each other. This persistence of form (or *covariance*, as it is technically called) in the differential equations that express the laws of nature does not in general imply that the actual *magnitudes* measured by two observers in uniform relative motion will be the same.

E.g., an observer moving with his instruments relative to an electrically charged body will detect magnetic as well as electrical forces, whilst one who is at rest with his instruments relative to this body will observe only electrical forces. But the differential equations connecting the effects noted by one observer with each other and with his x, y, z, and t co-ordinates will be precisely the same as those connecting the effects noted by the other observer with each other and with *his* x, y, z, and t co-ordinates.

This principle by itself, however, would be of little use, since it does not enable us to say what connexion exists between the co-ordinates of the two observers. But, if there be some physical magnitude, which is not merely covariant but also *invariant* as between different observers in uniform relative motion, the transformations connecting the two sets of co-ordinates can be found. Now the velocity of light *in vacuo* is found to fulfil this condition; its actual numerical value is found to be the same by all observers. The mathematical consequences of this fact lead to Einstein's set of equations connecting the x, y, z, t co-ordinates of one observer with those of another who is moving relative to the first. The precise significance of Einstein's principle of the 'Constancy of Light Velocity' has been indicated above, and shown to be connected with the way in which we are forced to lay out a system of co-ordinates and to define simultaneity between events in different places.

Einstein's restricted theory has gained many triumphs. It explains at once what is known as Fresnel's dragging-coefficient for light passing through matter that moves relative to the observer. It also accounts for the change of mass with velocity which is observed when small particles move with speed comparable to that of light. The principle necessitates slight changes in the previously accepted form of some of the laws of nature. Maxwell's equations and the equation of continuity in hydrodynamics do indeed at once and without modification fulfil the condition of covariance. But the laws of mechanics, as they stand, are not in accord with the principle and need modifications which only become practically important in dealing with the motion of matter with velocities comparable to that of light.

Considerable philosophic importance, in connexion with the nature of time, attaches to the work of Minkowski.

On the ordinary Newtonian mechanics the form of the laws of nature is unchanged if the three spatial axes be twisted in space about their origin as a rigid body. Now Minkowski showed that the Lorentz-Einstein transformation is equivalent to a twist of the same nature performed on a set of *four* mutually rectangular axes in a four-dimensional space. Three of these axes are the ordinary spatial ones, the fourth is the time axis multipled by c, the velocity of light, and ι, the root of -1. So far the theory must be regarded as a merely elegant mathematical device, since the fourth axis is imaginary in the mathematical sense, and the angle of solution is also imaginary. But, if we do not assume that the geometry of the four-dimensional 'space-time' is Euclidean, a much more important meaning can be attached to Minkowski's interpretation of the relativity transformations. If we suppose the geometry of 'space-time' to be hyperbolic (*i.e.* the geometry of Lobatchewski), the relativity transformation corresponds to twisting a set of four real axes as a rigid body through a real angle about the origin. The axes are now x, y, z, and ct, and c simply depends on the different units that we use in measuring time and space; so that really we are dealing with a four-dimensional

[1] This is the famous Lorentz-Fitzgerald contraction.

manifold in which space and time are homogeneous with each other, but whose geometry is not Euclidean but Lobatchewskian.

The work of philosophical mathematicians since Minkowski's death has consisted largely in developing the notion that the ultimate data in the world are events in space-time, *i.e.* events extended both in space and in time. The content of a specious present forms an example of such data. Space and time as used in the sciences only emerge at the end as elaborate mathematical constructions built on the immediately perceptible relations between extended events.

The best exposition of this point of view is contained in A. N. Whitehead's *Principles of Natural Knowledge*, which begins with a severe criticism of the concepts of classical physics and proceeds to elaborate the notions of space, time, and matter from the crude data of sense and their immediately given spatio-temporal relations. It seems hardly possible to doubt that this is the right path for further research, but it demands a combination of philosophical and mathematical abilities of so high an order that few can tread it successfully. Alexander in his *Gifford Lectures* at Glasgow has developed the notion of space-time with great fullness from the purely philosophic side, but, at the time of writing, his lectures have not appeared in print, and it is impossible to give a fair account of his views from the short synopses which are alone available.

It remains to say a few words about the generalized Theory of Relativity. So far we have only considered observers in *uniform* relative motion and have laid down a principle of relativity for them. Einstein has occupied himself in the last few years in removing this restriction and thus bringing gravitation, which fell outside the older theory, into the scope of the Theory of Relativity.

A particle is said to be under the action of no force if it move uniformly in a straight line. But the question whether it moves in a straight line and whether it moves uniformly is clearly relative to our spatial axes and to our measure of time. If, *e.g.*, a particle moves uniformly in a straight line relative to the rectangular axes x and y, it will not do so relative to axes which rotate about the origin in the xy plane. Accordingly, relative to one set of axes it will be said to be under the action of no force, whilst relative to the second set it will be said to be under the forces needed to produce the observed accelerations. Now the 'forces' introduced by these mere changes of our axes of reference are in one respect very much like the force of gravitation. They, like it, affect all forms of matter indifferently and depend only on the mass, not on the special nature of the matter. On the other hand, a genuine gravitational field cannot be altogether transformed away by a suitable change of axes, as a purely geometrical field can be. For any one particle this can be done by choosing axes fixed in the particle, but relative to these axes the other particles in the field will still be accelerated. Now it seems clear that a mere change of axes could not make any difference to the form of the laws of nature, and thus, if gravitation were capable of being transformed away merely by a suitable change of axes, the principle of relativity would assert that the presence of a gravitational field makes no difference to the form of the laws of nature. For the reason mentioned above the principle of relativity cannot be taken in this unrestricted sense. It may, however, be taken to assert that the form of the law of nature is unaltered in a gravitational field up to a certain (as yet undetermined) order of differential coefficients.

It is now necessary to see the bearing of these results on the constitution of the 'space-time' of nature.

It has been proved by Riemann that the metrical geometry of any space is completely determined when the 'linear element,' *i.e.* the interval between any pair of infinitely near points, is expressed as a known function of the differentials of the co-ordinates. Thus a three-dimensional Euclidean space is completely defined by the equation

$$ds^2 = dx^2 + dy^2 + dz^2$$

for the linear element. Now the metrical properties of four-dimensional space-time will be completely determined when ds^2 (the interval of any pair of adjacent points in it) is expressed as a known function of dx^2, dy^2, dz^2, dt^2, $dxdy$, $dxdz$, $dxdt$, . . . etc. In space-time, therefore, the ten coefficients of dx^2, dy^2 . . . must be known in order to determine ds^2. In general these coefficients will be functions of x, y, z, t; they are denoted by the letters g_{xx}, g_{yy}, g_{xy}, etc. Any transformation of axes corresponds to a change in these g's and therefore to a change in the form of the linear element. It follows that, as regards forces introduced simply by changes of axis, it is a matter of perfect indifference whether we say (*a*) that the geometry of space-time is such and such and that such and such forces are acting, or (*b*) that the geometry of space-time is such as to produce the appearance of these forces. The g's can be regarded either (*a*) as completely determining the forces on a given assumption about the geometry of space-time; or (*b*) as determining the metrical properties of space-time itself. The extended Theory of Relativity prefers to take the second view of them and to drop all reference to forces; on the first view the g's are of the nature of potentials. Now, in theory, any function whatever might be chosen for the g's. But, in fact, all parts of nature are subject to gravitation. This means that the choice of g's is not absolutely unrestricted, but that in every permissible system of axes for describing nature the g's will be subject to a set of differential equations connecting them with each other and with the x, y, z, and t of that system. These equations then express the law of gravitation and at the same time express it as a fundamental property of space-time.

It is extremely difficult to render Einstein's theory intelligible without mathematics, and the mathematics needed is somewhat formidable. It is hoped, however, that the above slight sketch may illustrate that extreme entanglement of time with space and with matter which undoubtedly occurs in our crude sense-data and is now seen to persist even in the most refined speculations of mathematical physics. It may perhaps be added that Einstein's generalized theory, as distinct from the special philosophic interpretations which may be put on it, is not a mere idle speculation, but has already explained the anomalies in the perihelion of Mercury, and has correctly foretold the amount of deviation in a ray of light due to its passing near a heavy body like the sun.

8. Historically important speculations about time.—Our knowledge of time as of space owes more to the labours of mathematicians and physicists than to those of professed philosophers. The sharp distinction between time and what changes, and between space and what moves in it and is extended, is largely due to the development, first of mechanics, and latterly of electrodynamics.

To the Greeks we owe much less with regard to time than with regard to most matters of philosophic or scientific speculation. This may perhaps be ascribed to the late development of dynamics; the Greek approach to the problems of time was mainly by way of astronomy. Of course, Zeno's celebrated arguments have an important bearing on change and continuity, and, whatever may have been the real intention of their author, they remained the best discussion on these subjects so closely related to time until the final treatment of infinity and continuity by Dedekind and Cantor in the latter part of the 19th century. Time plays an important part in the *Timæus* of Plato; and, although his treatment cannot be called satisfactory, it has the merit of distinguishing time from what is in time.

Plato says that God wished the created world to resemble the intelligible one as far as possible. Now, it was not possible for it to be eternal, and the nearest analogue to eternity which He could provide was to make 'a moving image of eternity.' This is time, and it is closely connected with the motions of the heavens; eternity 'rests in unity,' but the image 'has a motion according to number.' Before the heavens were created, there were no days, years, etc.; but, when God created the heavens, He created these divisions of time also. Time was thus created with the heavens, and, if one were to be dissolved, so would the other be. But Plato does not appear to identify time with the motion of the heavens, though it is difficult to see what he supposes it to be in itself. According to Plato, past and future are created species of time which we wrongly transfer to the eternal essence; strictly 'was' and 'will be' are to be asserted only of generation in time, for they are motions. The analogy of the moving image to the eternal is that the created heavens have been, are, and will be in all time. This view has something in common with that of Spinoza, who makes things as they really are for *ratio* timeless, but holds that this timelessness cannot be grasped by *imagination*, which represents it confusedly as duration through endless time.

Aristotle defines time as 'the number of motions relative to before and after.' Number here appears to mean what is numbered. The now is borne along with the movable as a point may be regarded as moving and making up a line. So in a sense there is only one now, though in another sense there are many nows. This is obviously a very unsatisfactory metaphor, and there seems no reason to think that Aristotle was really clear as to the distinction between time and motion.

The Schoolmen in the main adopted Aristotle's views, though with certain modifications. St. Thomas Aquinas, in the tract *de Instantibus*, discusses time and change with some fullness. He draws a distinction between the time in which angels perform their acts and that in which men and matter operate. The time of angels is discrete, that of men continuous; the difference arises from the fact that continuity is essentially connected with matter, while angels are separated substances. An instant for an angel is the time occupied by a single act; it may thus correspond to a long period in our time. This may be compared with Royce's views about the varying lengths of the specious present in various beings.

In modern philosophy the men who have most concerned themselves with time are Leibniz and Kant. Leibniz argued strongly for the relative view of time in his letters to Clarke, who represented Newton and the absolute theory. His arguments turn mainly on the identity of indiscernibles and the principle of sufficient reason. Leibniz carefully distinguished duration from the relation of before and after, and he compared duration to the extension of matter. Leibniz's view is that time is a system of possible positions of possible events related by before, after, and simultaneous with. He holds that all possible worlds must be in time, though, of course, the particular temporal relations of the actual world are contingent. To make Leibniz's theory coherent, it would be necessary to be much clearer than he is as to the relation between the time-series of each monad and the time-series of the universe. He attempted to explain the relation between successive states of the same monad by saying that the earlier ones have the quality of being desires for the later ones. As an attempt to replace relations by qualities this clearly fails, since 'desire for' anything is clearly a disguised relation. And as an attempt to *define* before and after it also fails; for it is clearly a synthetic proposition that desire for X precedes X. Then again it seems essential to Leibniz's doctrine of the reflexion by one monad of the states of another that we should have some account of the temporal relations between corresponding states in different monads. The state of a monad at a given moment in its own time-series is presumably the reflexion of the *contemporary* states of other monads; but we are not told what is meant by a time-series common to the monads, nor is it clear that this would be consistent with Leibniz's dislike of relations.

The absolute theory of time has never had much philosophic support; there can be little doubt that Leibniz had the better of Clarke. Perhaps the best arguments for absolute time and space are to be found in Bertrand Russell's *Principles of Mathematics*. They do not seem to the present writer to be conclusive, and their author has latterly taken a much more relativistic view.

Locke, Berkeley, and Hume insisted that the notion of time comes from the succession of our ideas. But they never made it clear how their temporal relations are connected with the time that is used in physics. Berkeley and Hume in particular fail to give any reasonable account of the distinction that we certainly make between the temporal order of our ideas and the temporal order of the objects which we claim to know by them. It is a great merit of Kant to have seized on the importance of this point in his 'analogies of experience,' though the distinction will certainly not bear the superstructure which he built on it. He attempted to prove that the distinction involves the permanence of substance (which he seems to identify with the chemical law of the conservation of mass) and the law of causation among experienced objects. But his arguments are entirely inconclusive even to prove that, in order to make the distinction, we must *believe* in these principles; much less to prove, what the transcendental method always tends to confuse with this, that the principles are true.

Time plays perhaps more, and more important, parts in Kant's philosophy than in any other.

(1) In the *Æsthetic* he tries to prove that it is a form of intuition, the form appropriate to the internal sense. This seems to mean that, just as we can only perceive physical objects as being in space, though there is no reason to think that things-in-themselves are spatial, so we can only perceive ourselves and our mental states in introspection as being in time, though there is no reason to think that we really are in time. This certainly seems to raise the special difficulty that, unless we know ourselves as we are and not merely as we appear, we cannot know what our forms of intuition are, but only what they appear to be, whilst Kant's argument certainly assumes that we know what they are. (2) In the *Dialectic*, as we have seen, Kant has an antinomy about time. This apparently would, if valid, overthrow not merely absolute time but also the temporal character of events and the temporal relations between them. We have already seen how grave are the difficulties in the way of any such conclusion, and how entirely powerless Kant's arguments are to prove it. (3) In the *Analytic* time plays an important part in the difficult doctrine of the schematism of the categories. The position seems to be that the categories as pure conceptions of the understanding cannot be applied immediately to the manifold given in sense, even after that has been synthesised by imagination. They have to be mediated through time; thus the category of ground and consequent, which is purely logical, can be applied to the world of sensible experience only after it has been schematised into the temporal form of cause and effect. The whole argument here is confused and weak to a remarkable degree; the principle appears to be that the manifold of sense is provided with temporal characteristics by intuition; that these remain and are elaborated by the syntheses of imagination; and that then the categories can be applied if they be first schematised so that they and the synthesized manifold share the temporal characteristic in common. (4) Kant's critical solution of his own antinomy is that the infinity involved in time is not an actual infinite, as it would have to be if time applied to things-in-themselves, but is only the power that we have of always synthesising farther than we have yet gone in constructing a temporal series. To this Lotze makes the very pertinent criticism that it surely depends on the nature of things-in-themselves whether we shall be indefinitely supplied with material to synthesise.

The modern development of our knowledge about time is due mainly to two sets of people: (1) philosophical mathematicians, like Dedekind and Cantor, who have given a satisfactory analysis of infinity and continuity, and thus finally refuted all antinomies based on these; (2) mathematical physicists who have been led by their studies in the optics of moving systems to elaborate the Theory of Relativity. The pioneer in this work is Lorentz; the theory itself was first formulated by Einstein; and the mathematical and philosophical consequences have been drawn and elaborated by Einstein, Minkowski, Robb, Whitehead, and others.

It is also necessary to mention among recent philosophers Bergson, in whose works time, nominally at any rate, plays an important part. Bergson holds that the attempt to treat time as similar to space is a perverse one philosophically; it may work very well in dealing with dead matter, but it shows its falsity in biology, psychology, and philosophy. He also falls foul of the mathematical theory of the *continuum* as applied to time; he admits that it is internally consistent, but denies that it describes what anybody really means by change and motion. Bergson's arguments seem to rest partly on a comparison between change as a sense-datum (*e.g.*, the peculiar characteristic of what we see when we look at the second hand of a watch as distinct from the hour hand) and physical change, and partly on the erroneous view that a whole of related states cannot be a change unless each of its terms be a change. Again, in some of his remarks about memory he seems to suppose that, because a memory-act is a later awareness of an earlier event, the earlier event and the later awareness must somehow be contemporary. Finally, he seems to think that the ordinary view of time is refuted by the facts, of which he is strongly convinced, that no two total states of mind at different times can be exactly alike, that there are not, strictly speaking, distinct elements which can recur as parts of different mental states, and that no amount of knowledge about earlier states will enable us to foretell later ones completely. But Bergson's most characteristic doctrines belong to the subject of change rather than to that of time.

LITERATURE.—(*a*) *Historical.*—Plato, *Timæus*; Aristotle, *Physics*; St. Thomas Aquinas, *de Instantibus.* (*b*) *Absolute and relative theories.*—Leibniz, *Letters to Dr. Samuel Clarke*, London, 1917; Bertrand Russell, *The Principles of Mathematics*, Cambridge, 1903, i. (*c*) *Reality of time.*—Leibniz; Kant, *Critique of Pure Reason*; F. H. Bradley, *Appearance and Reality*, 2nd ed. revised, London, 1902; J. Ellis McTaggart, 'The Unreality of Time,' *Mind*, new ser., xvii. [1908] no. 68, p. 457 ff.; H. Bergson, *Time and Free-will*, Eng. tr., London, 1910, and *Matter and Memory*, Eng. tr., do. 1911. (*d*) *Measurement of time and Theory of Relativity.*—H. Poincaré, *Science et méthode*, Paris, 1908, and *La Valeur de la science*, do. 1908; H. A. Lorentz, *The Theory of Electrons*, London, 1909; H. Minkowski, *Raum und Zeit*, Leipzig, 1909; L. Silberstein, *The Theory of Relativity*, London, 1914; C. D. Broad, *Perception, Physics, and Reality*, Cambridge, 1914; A. A. Robb, *A Theory of Time and Space*, London, 1911; A. S. Eddington, *Report on the Relativity Theory of Gravitation*, do. 1918. (*e*) *Physical time as a construction.*—B. Russell, *Our Knowledge of the External World*, London, 1914; A. N. Whitehead, *Principles of Natural Knowledge*, Cambridge, 1919.

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TINNEH.—See DÉNÉS.

TIPITAKA.—See LITERATURE (Buddhist), vol. viii. p. 85[b].

TIRUPATI.—Tirupati, vulg. Tripetty (Tel. Tirupati, *tiru*, Skr. *śrī*, 'venerable,' *pati*, 'lord'), a town in Chittoor District, Madras (lat. 13° 38′ N., long. 79° 24′ E.), is a famous place of pilgrimage, situated on the Tirumalai or sacred hill, usually known to Europeans as Upper Tirupati, in contrast to the lower town at its base. The whole area is considered sacred, and up to 1870 had never been visited by Europeans. Mark Wilks states that he was on duty for eighteen months in the neighbourhood, and, though he frequently climbed the adjoining hills, he could never catch even a distant view of the pagoda.[1] The belief that much crime was committed without detection in the holy town led to the issue of an order by Government that it should be thrown open to the District officials. This at first produced considerable local opposition; but European visits now cause little sensation. The sanctity of this hill-range rests on the legend that it forms part of the sacred mountain Meru. The range has seven principal peaks, each of which is sacred and has a name and legend of its own. One of the peaks, known as Seshāchalam, 'serpent hill,' takes its name from the belief that it was torn from Meru by Ādi Sesha, the primordial snake, who contended in a trial of

[1] *Hist. Sketches of the South of India*[2], Madras, 1869, i. 246 n.

strength with the wind-god, Vāyu. Vāyu raised so great a tempest that the peak was blown away and fell to earth in its present position. Near this peak the great temple stands. Little can be seen of it, and no European has been allowed to enter it. It is a building of little architectural beauty or importance, but the cultus of the deity is interesting as an example of the amalgamation of local non-Aryan beliefs with orthodox worship. Within a small chamber lighted by lamps is the idol, a stone image of Viṣṇu, seven feet in height. It represents the god as Chaturbhuja, 'four-armed,' one of the right hands holding the discus (*chakra*), one of the left the conch-shell (*sankha*), the second right hand pointing to the earth to draw attention to the miraculous origin of the holy hill, while the remaining left hand grasps a lotus. The deity possesses 1008 titles, the most common of which are Śrīnivāsa, 'dwelling with Śrī or Lakṣmī,' goddess of prosperity, and Venkaṭachalapati, the title of the sacred hill, which has been adopted into Sanskrit from the Tamil *ven*, 'white,' *kadam*, 'hill slope,' thus showing that the deity was adopted into Brāhmanism from a Dravidian cult. By visitors from the Deccan and N. India he is generally known as Bālāji, which, according to Monier-Williams,[1] is the name of a human incarnation of Viṣṇu or Kṛṣṇa of whom little is known, save that he was remarkable for many extraordinary qualities, and that he lived in the neighbourhood of the sacred hill. Hence visitors to the shrine generally invoke him by the title of Govinda, 'cow-keeper,' one of the names of Kṛṣṇa. It is remarkable, however, that, according to common belief, the image worshipped was originally one of Śiva. The transformation of the Śaiva cult to that of Viṣṇu is traditionally ascribed to the reformer Rāmānujāchārya (born *c.* A.D. 1017). It is said that he procured a conch-shell and discus of gold, which he placed before the image and closed the temple doors. When the shrine was opened next day, it was found that these emblems of Viṣṇu were grasped in the hands of the image, and therefore it was really Viṣṇu. The tangled hair (*jaṭa*), the cobras carved upon the body, and various other peculiarities indicate that it was intended to represent Śiva, and the priests, who are Dīkṣhita Brāhmans, admit that they belong to the Śaiva sect. The god is provided with a consort, Padmāvatī, said to be the incarnation of a mortal woman, and the offerings are believed to have been originally collected to provide for the marriage of the pair. In an ante-room there is a brass vessel with a bag hanging in it, into which money and jewels are placed. On the other side are two gongs, one of which, when struck, utters the name Govinda, the other Nārāyaṇa—both titles of the god. Many pious persons observe the custom of collecting in their homes monthly contributions which are placed in a money-box and finally offered at the shrine.[2] The anthropomorphism of the cult is shown in the belief that the deity annually announces to certain persons that he needs shoes, which they make and present.[3] Various rites indicate the non-Aryan character of the worship. Thus a feast called *Gangājātra*, 'Ganges festival,' is held in the early spring, when a figure is made of clay or straw, before which animals are sacrificed—a custom quite opposed to true Vaiṣṇava beliefs. Even Brāhmans, who will not attend personally, send victims. When the sacrifices are over, the image is burned, and much rude merriment follows. Some votaries carry on their heads a structure made of bamboo, resembling a car, adorned with coloured paper, and supported by iron nails that

[1] *Brāhmanism and Hindūism*[4], London, 1891, p. 267 f.

[2] E. Thurston, *Ethnographic Notes in S. India*, Madras, 1906, p. 352.

[3] Thurston, *Castes and Tribes of S. India*, iv. 310 f.